boundary layer with external flow field pressure gradients. Massachusetts Institute of Technology. Naval Supersonic Laboratory, Tech. Rep. 419, (1959).
5. D. G. HURLEY, Mass transfer cooling in a boundary layer, *Aeronaut. Quart.* **XII**, 165–188 (1961).
6. J. F. GROSS, J. P. HARTNETT, D. J. MASSON and C. GAZLEY, JR, A review of binary laminar boundary layer characteristics, *Int. J. Heat Mass Transfer*, 3, 198–221 (1961).
7. J. P. HARTNETT and E. R. G. ECKERT, Mass transfer cooling in a laminar boundary layer with constant fluid properties, *Trans. Amer. Soc. Mech. Engrs* **79**, 247–254 (1957).
8. J. R. BARON, Thermodynamic coupling in boundary layers. *J. Amer. Rocket Soc.* **32,** 1053–1059 (1962).
9. O. E. TEWFIK, E. R. G. ECKERT and C. J. SHIRTLIFFE, Thermal diffusion effects on energy transfer in a turbulent boundary layer. Proceedings, 1962 Heat Transfer and Fluid Mechanics Institute, Stanford University Press, 42–61 (1962).

TURBULENT HEAT TRANSFER IN A PARALLEL-PLATE CHANNEL

E. M. SPARROW and S. H. LIN
Heat Transfer Laboratory, University of Minnesota

(*Received* 3 *August* 1962)

THE purpose of this brief paper is to present analytical results for the fully developed heat-transfer characteristics of a turbulent flow in a parallel-plate channel with uniform wall heat flux. The results were obtained by integrating the energy equation utilizing a suitably chosen eddy diffusivity for heat. The analytical method parallels a prior study for the circular tube [1], and because of this, the details of the analysis will be omitted here. The circular tube results of [1] have received strong support from experiment (e.g. [2] and [3]). It is this good agreement which has prompted the present extension of the analysis to the parallel-plate channel. Friction factor results will also be given here.

The fully developed Nusselt numbers are presented in Fig. 1 as a function of Prandtl number for the range $Pr = 0{\cdot}7$ to 100. The Reynolds number, which appears as parameter, ranges from 10 000 to 500 000. The hydraulic diameter, D_e, is equal to twice the spacing between the plates; while the heat-transfer coefficient h is the ratio of the local heat flux to the local wall-to-bulk temperature difference. $\bar{U}$ is the mean velocity.

The results of the present analysis appear as solid lines on the figure. Also shown on the figure are dashed lines representing McAdams' [4] empirical correlation of circular tube data

$$Nu = 0{\cdot}023\, Re^{0{\cdot}8}\, Pr^{0{\cdot}4}. \qquad (1)$$

This correlation is usually regarded as applying as a first approximation to non-circular geometries provided that the hydraulic diameter is used. In addition, there are dot-dashed curves which represent the analytically-determined circular tube results of [1]; for $Re = 10^4$, only a single point was available and this is symbolized on the figure by a blackened circle. Finally, there is shown by triangles Deissler's [5] analytical findings for the parallel plate channel, available only for $Pr = 0{\cdot}73$. Aside from the latter, the authors are unaware of other analytical results for turbulent flow in a parallel-plate channel.

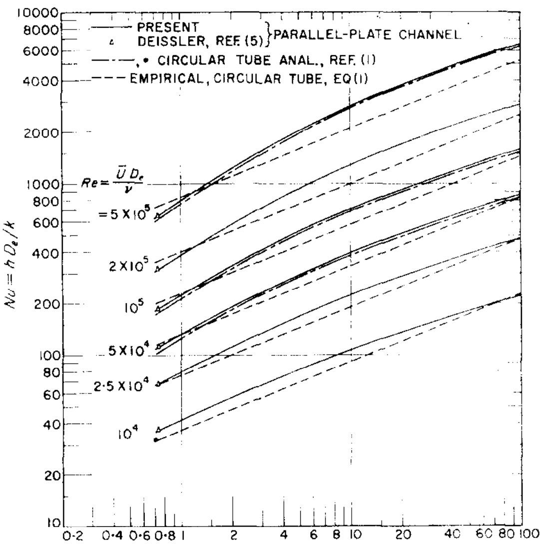

FIG. 1. Nusselt number results.

Comparison of the solid and dot-dashed curves indicates that the use of the hydraulic diameter is reasonably successful in bringing together the analytical results for the two geometries. Additionally, Deissler's results for $Pr = 0{\cdot}73$ are in good agreement with those of the present analysis.

The analytical results for both geometries (solid and dot-dashed curves) suggest that the dependence of the Nusselt number on Prandtl number is more complex than the simple power law appearing in the empirical correlation. Deissler [6] has shown a similar finding for the circular tube. The largest deviations between the empirical correlation and the theory appear in the mid-range of Prandtl numbers ($\sim$10). However, recent careful measurements by Allen [3] for water flowing in a circular tube have demonstrated 1 per cent agreement with the predictions of [1] at $Pr = 8$. The experiments were carried out so as to eliminate the effects of variable fluid properties. Allen's work lends strong support to the analytical model.

Friction factor results for fully developed flow conditions are presented in Fig. 2. The solid line represents the present results for the parallel-plate channel; while the dot-dashed line is the corresponding analytical prediction for the circular tube. The dashed line corresponds to an empirical correlation of circular tube data by Moody [7]. The τ_w appearing in the ordinate variable is the shear stress at the wall; this is simply related to the pressure drop by means of a momentum balance. Considering first the analytical curves, it is seen that the use of the hydraulic diameter is reasonably effective in bringing together the results for the two geometries, with those for the circular lying 4–6 per cent below those for the parallel-plate channel. The agreement between the analytical and empirical results for the circular tube is better at higher Reynolds numbers and is generally quite satisfactory.

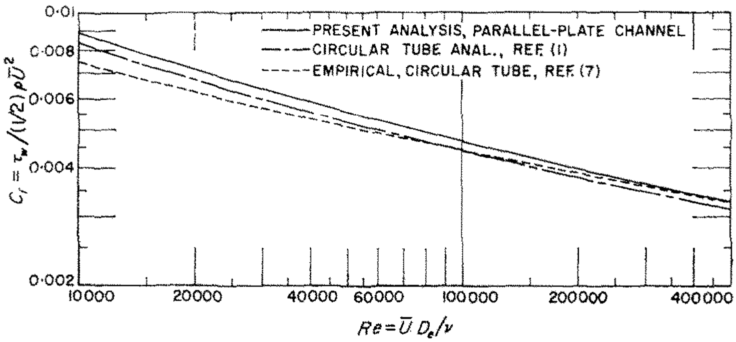

FIG. 2. Friction factor results.

ACKNOWLEDGEMENT

The support of the U.S. Atomic Energy Commission under contract AT(11-1)-659 is gratefully acknowledged.

REFERENCES

1a. E. M. SPARROW, T. M. HALLMAN and R. SIEGEL, Turbulent heat transfer in the thermal entrance region of a pipe with uniform heat flux, *App. Sci. Res.* A7, 37 (1957).

1b. R. SIEGEL and E. M. SPARROW, Comparison of turbulent heat transfer results for uniform wall temperature and uniform wall heat flux, *J. Hea Transfer*, **82**, 152 (1960).

2. R. SIEGEL and E. M. SPARROW, Turbulent flow in a circular tube with arbitrary internal heat sources and wall heat transfer, *J. Heat Transfer*, **81**, 280 (1959).

3. R. W. ALLEN, Measurements of friction and local heat transfer for turbulent flow of a variable property fluid in a uniformly heated tube. Ph.D. Thesis, University of Minnesota, Department of Mechanical Engineering (1959).

4. W. H. MCADAMS, *Heat Transmission* (Third Ed.), p. 219. McGraw-Hill, New York (1954).

5. R. G. DEISSLER, Analysis of turbulent heat transfer and flow in entrance region of smooth passages, *NACA* TN 3016 (1953).

6. R. G. DEISSLER, Analysis of turbulent heat transfer, mass transfer, and friction in smooth tubes at high Prandtl numbers. *NACA* Report 1210 (1955).

7. L. F. MOODY, Friction factors for pipe flow, *Trans. ASME*, **66**, 671 (1944).